# IIT–JEE 2004 SCREENING SOLUTIONS

Memory Based (Version 4)

**Correct answers are marked with *.** **Expected cut-off for screening: 100 marks out of 252 marks i.e. around 40%.**

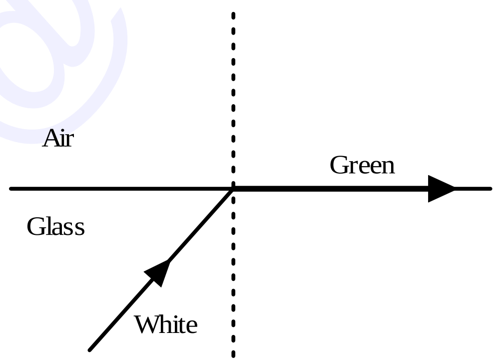
## PHYSICS

**Break-up of questions:**

| Mechanics | Sound | Heat | Electromagnetism | Optics | Modern Physics |
|---|---|---|---|---|---|
| 8 (29%) | 2 (7%) | 4 (14%) | 7 (25%) | 4 (14%) | 3 (11%) |

**Remarks: Mechanics + Electromagnetism constitute more than 50% as expected.**

**1.** White light is incident on the interface of glass and air as shown in the figure. If green light is just totally internally reflected then the emerging ray in air contains




*(a) yellow, orange, red

(b) violet, indigo, blue

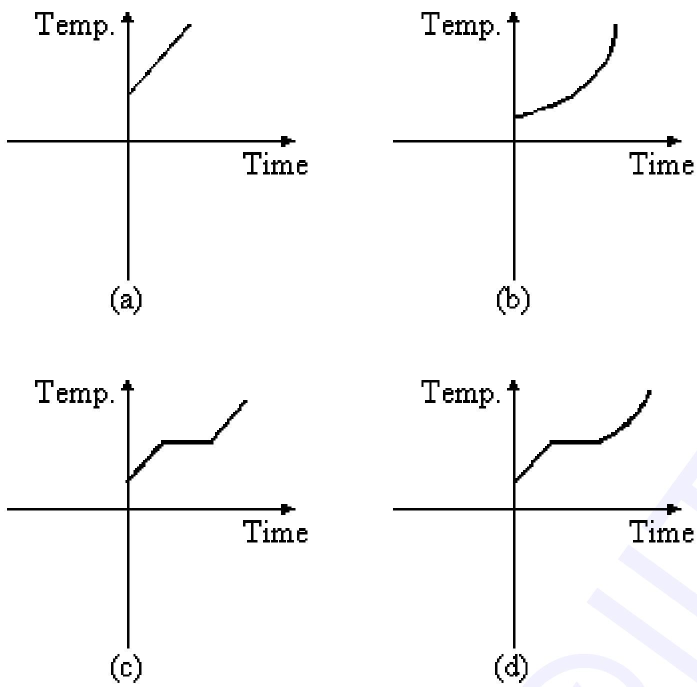
(c) all colours

(d) all colours except green

➢ $C = \sin^{-1}\left(\frac{1}{\mu}\right)$ and in the sequence VIBGYOR, $C$ increases.

The angle of incidence = critical angle for green colour.

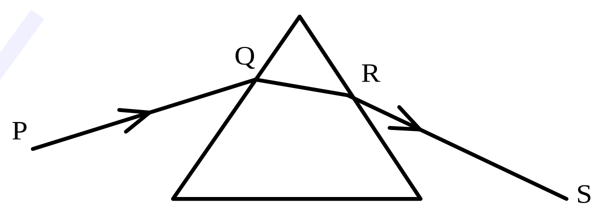
**2.** Liquid oxygen at 50 K is heated to 300 K at constant pressure of 1 atm. The rate of heating is constant. Which of the following graphs represents the variation of temperature with time?




➤ Liquid oxygen when heated from 50 K to 300 K will change its phase. During phase change the temperature does not change. After that (i.e. in gaseous phase) the temperature increases linearly if rate of heating is constant.

∴ (c)

**3.** A ray of light is incident on an equilateral glass prism placed on a horizontal table. For minimum deviation which of the following is true?




(a) *PQ* is horizontal

*(b) *QR* is horizontal

(c) *RS* is horizontal

(d) Either *PQ* or *RS* is horizontal

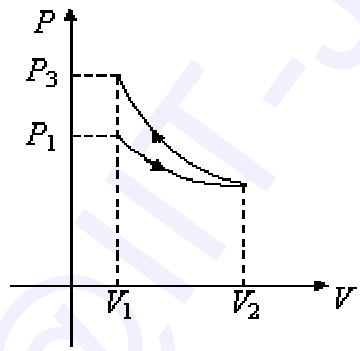
➤ When the prism is in the position of minimum deviation, i = e or the ray *inside* the prism is parallel to the base of the prism.

**4.** An ideal gas expands isothermally from a volume $V_1$ to $V_2$ and then compressed to original volume $V_1$ adiabatically. Initial pressure is $P_1$ and final pressure is $P_3$. The total work done is $W$. Then

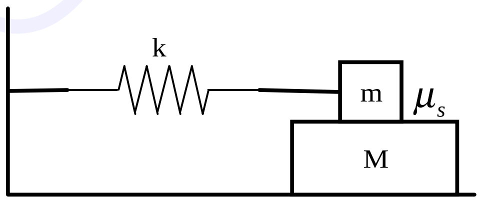
(a) $P_3 > P_1$, $W > 0$ (b) $P_3 < P_1$, $W < 0$

*(c) $P_3 > P_1$, $W < 0$ (d) $P_3 = P_1$, $W = 0$

➢ The two processes are shown in the adjoining *PV* - diagram.




**5.** A block *P* of mass *m* is placed on a frictionless horizontal surface. Another block *Q* of same mass is kept on *P* and connected to the wall with the help of a spring of spring constant *k* as shown in the figure. $\mu_s$ is the coefficient of friction between *P* and *Q*. The blocks move together performing SHM of amplitude *A*. The maximum value of the friction force between *P* and *Q* is




(a) $kA$ *(b) $\frac{kA}{2}$

(c) zero (d) $\mu_s mg$

➢ For block *P*, friction provides the restoring force.

$\therefore$ $f_{max} = m\omega^2 A$ with $\omega^2 = \frac{k}{m+m}$

**6.** After 280 days, the activity of a radioactive sample is 6000 dps. The activity reduces to 3000 dps after another 140 days. The initial activity of the sample in dps is

(a) 6000 (b) 9000

(c) 3000 *(d) 24000

➢ The half-life of the substance is 140 days. In 420 days, there will be three half-lives.

**7.** The energy of a photon is equal to the kinetic energy of a proton. The energy of the photon is $E$. Let $\lambda_1$ be the de-Broglie wavelength of the proton and $\lambda_2$ be the wavelength of the photon. The ratio $\lambda_1/\lambda_2$ is proportional to

(a) $E^o$ *(b) $E^{1/2}$

(c) $E^{-1}$ (d) $E^{-2}$

➢ $\lambda_1 = \frac{h}{\sqrt{2mE}}$ and $\lambda_2 = \frac{hc}{E}$

**8.** A closed organ pipe of length $L$ and an open organ pipe contain gases of densities $\rho_1$ and $\rho_2$ respectively. The compressibility of gases is equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency. The length of the open organ pipe is

(a) $\frac{L}{3}$ (b) $\frac{4L}{3}$

* (c) $\frac{4L}{3}\sqrt{\frac{\rho_1}{\rho_2}}$ (d) $\frac{4L}{3}\sqrt{\frac{\rho_2}{\rho_1}}$

➢ Use the equations $\frac{3v_c}{4L} = \frac{2v_o}{2l}$ and $\frac{v_o}{v_c} = \sqrt{\frac{\rho_1}{\rho_2}}$ and proceed.

**9.** In the relation

$$p = \frac{\alpha}{\beta} \exp\left( - \frac{\alpha}{k_b \theta} . z \right)$$

P is pressure, $Z$ is distance, $k_b$ is Boltzmann constant and θ is the temperature.

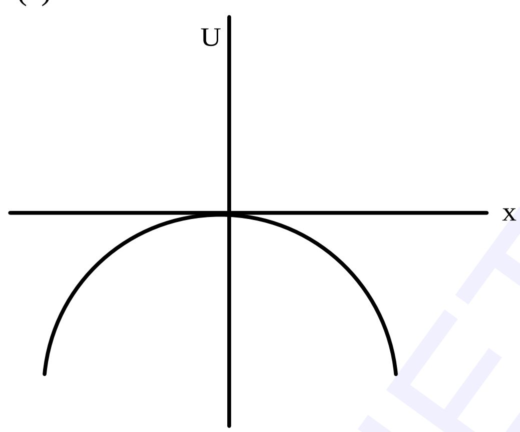
The dimensional formula of β will be

*(a) $[M^o L^2 T^o]$ (b) $[M^1 L^2 T^1]$

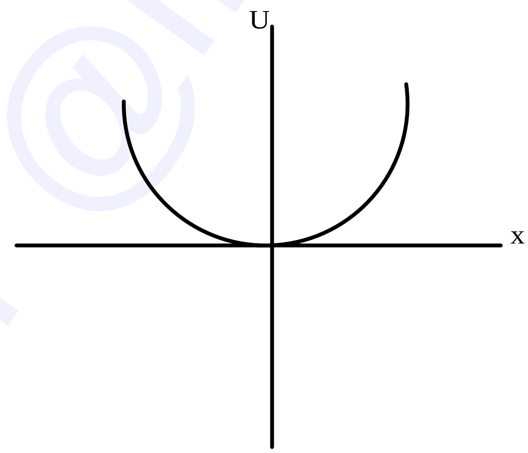
(c) $[M^1 L^o T^{-1}]$ (d) $[M^o L^2 T^{-1}]$

**10.** A particle is placed at the origin and a force $F = kx$ is acting on it (where $k$ is a positive constant). If $U(0) = 0$, the graph of $U(x)$ versus $x$ will be (where $U$ is the potential energy function)

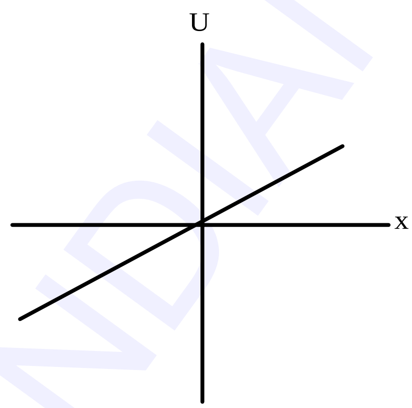
*(a)



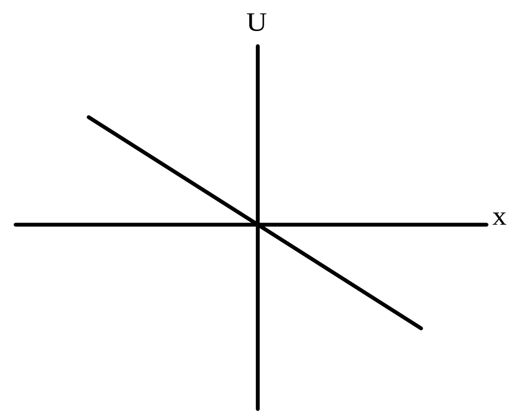

(b)




(c)



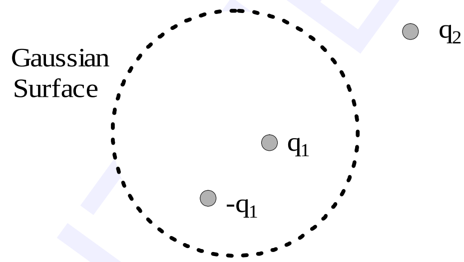

(d)



➢ $F = -\frac{dU}{dx} = kx$

Integrating, we have $U(x) = -\frac{1}{2}kx^2$

**11.** Consider the charge configuration and a spherical Gaussian surface as shown in the figure. When calculating the flux of the electric field over the spherical surface, the electric field will be due to



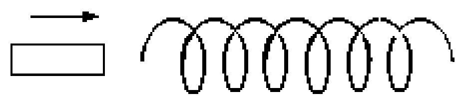

(a) $q_2$ (b) only the +ve charges

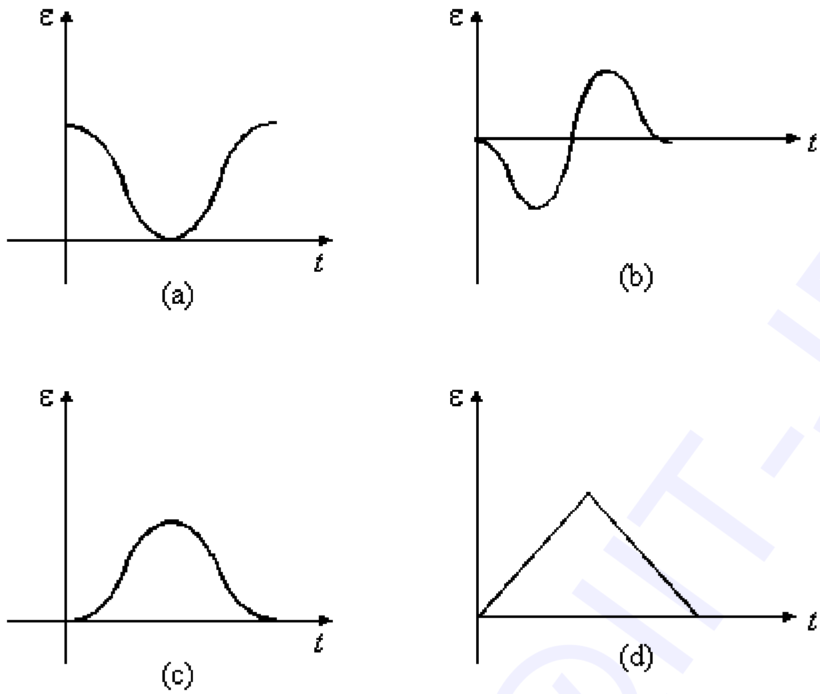
*(c) all the charges (d) $+q_1$ and $-q_1$

➢ In Gauss' law, the electric field is due to all the charges present whether inside or outside the Gaussian surface.

**Remarks: This question was discussed in the class.**

**12.** The variation of induced emf (ε) with time ($t$) in a coil if a short bar magnet is moved along its axis with a constant velocity is best represented as

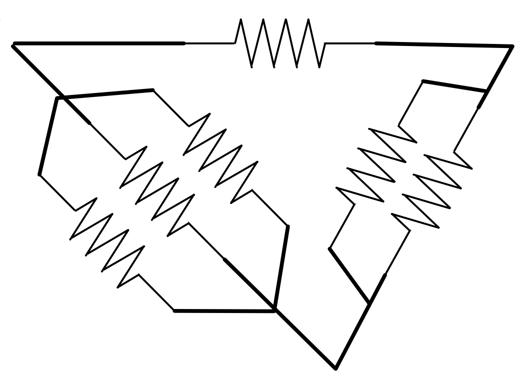





➢ As the magnet moves towards the coil, the magnetic flux increases (nonlinearly). Also there is a change in polarity of induced emf when the magnet passes on to the other side of the coil.

$\therefore$ (b)

**13.** Six equal resistances are connected between points *P*, *Q* and *R* as shown in the figure. Then the net resistance will be maximum between




*(a) *P* and *Q*

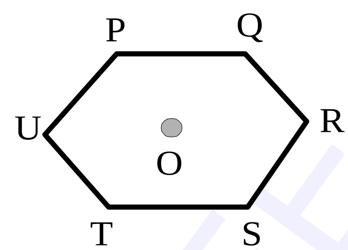
(b) *Q* and *R*

(c) *P* and *R*

(d) any two points

➢ When resistances are connected in parallel, the equivalent is less than the minimum resistance.

**14.** Six charges, three positive and three negative of equal magnitude are to be placed at the vertices of a regular hexagon such that the electric field at *O* is double the electric field when *only* one positive charge of same magnitude is placed at *R*. Which of the following arrangements of charges is possible for *P*, *Q*, *R*, *S*, *T* and *U* respectively?




(a) +, –, +, –, –, + (b) +, –, +, –, +, –

(c) +, +, –, +, –, – *(d) –, +, +, –, +, –

➢ If the charges are arranged according to the option (d), the electric fields due to *P* and *S* and due to *Q* and *T* add to zero, while due to *U* and *R* will be added up.

**15.** A source of sound of frequency 600 Hz is placed inside water. The speed of sound in water is 1500 m/s and in air it is 300m/s. The frequency of sound recorded by an observer who is standing in air is

(a) 200 Hz (b) 3000 Hz

(c) 120 Hz *(d) 600 Hz

➢ The frequency, a characteristic of source, is independent of the medium.

**16.** A point object is placed at the center of a glass sphere of radius 6 cm and refractive index 1.5. The distance of the virtual image from the surface of the sphere is

(a) 2 cm (b) 4 cm

*(c) 6 cm (d) 12 cm

➢ The rays from the object fall normally on the surface of the sphere and emerge undeviated.

**17.** In a Young's double slit experiment bi-chromatic light of wavelengths 400 nm and 560 nm are used. The distance between the slits is 0.1 mm and the distance between the plane of the slits and the screen is 1 m. The **minimum** distance between two successive regions of complete darkness is

(a) 4 mm (b) 5.6 mm

(b) 14 mm *(d) 28 mm

➢ Let $n$th minima of 400 nm coincides with $m$'th minima of 560 nm then

$$(2n-1)400 = (2m-1)560$$

$$\Rightarrow \quad \frac{2n-1}{2m-1} = \frac{7}{5} = \frac{14}{10} = \frac{21}{15}$$

i.e. 4th minima of 400 nm coincides with 3rd minima of 560 nm.

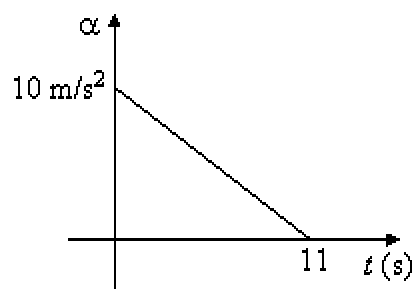
The location of this minima is = $\frac{7(1000)(400\times10^{-6})}{2\times0.1} =$ 14 mm

Next , 11th minima of 400 nm will coincide with 8th minima of 560 nm

Location of this minima is = $\frac{21(1000)(400\times10^{-6})}{2\times0.1} =$ 42 mm

$\therefore$ Required distance = 28 mm

**18.** A particle starts from rest. Its acceleration ($\alpha$) versus time ($t$) is as shown in the figure. The maximum speed of the particle will be




(a) 110 m/s *(b) 55 m/s

(c) 550 m/s (d) 660 m/s

➢ The area under the acceleration time graph gives change in velocity.

**19.** Two identical conducting rods are first connected independently to two vessels, one containing water at 100°C and the other containing ice at 0°C. In the second case, the rods are joined end to end and connected to the same vessels. Let $q_1$ and $q_2$ g/s be the rate of melting of ice in the two cases respectively. The ratio $q_1/q_2$ is

(a) $\frac{1}{2}$ (b) $\frac{2}{1}$

*(c) $\frac{4}{1}$ (d) $\frac{1}{4}$

➢ The rate of the melting of the ice will be inversely proportional to the equivalent thermal resistance between the two vessels.

In the first case, equivalent thermal resistance is $\frac{R}{2}$ and in the second case it is $2R$. (where $R$ is the thermal resistance of the rod).

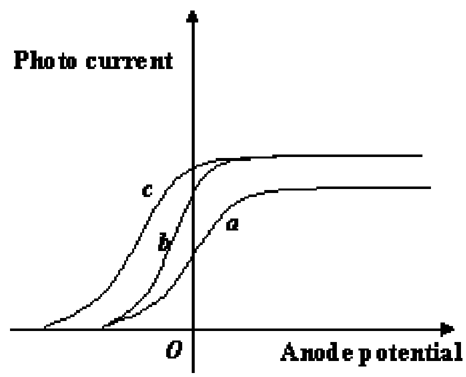
**20.** Three discs $A$, $B$, and $C$ having radii 2 m, 4 m and 6 m respectively are coated with carbon black on their outer surfaces. The wavelengths corresponding to maximum intensity are 300 nm, 400 nm and 500 nm respectively. The power radiated by them are $Q_A$, $Q_B$ and $Q_C$ respectively.

(a) $Q_A$ is maximum *(b) $Q_B$ is maximum

(c) $Q_C$ is maximum (d) $Q_A = Q_B = Q_C$

➢ $Q \propto AT^4$ and $\lambda_m T =$ constant $\Rightarrow$ $Q \propto \frac{r^2}{(\lambda_{max})^4}$

**21.** The figure shows the variation of photocurrent with anode potential for a photo-sensitive surface for three different radiations. Let $I_a$, $I_b$ and $I_c$ be the intensities and $f_a$, $f_b$ and $f_c$ be the frequencies for the curves $a$, $b$ and $c$ respectively.




*(a) $f_a = f_b$ and $I_a \neq I_b$

(b) $f_a = f_c$ and $I_a = I_c$

(c) $f_a = f_b$ and $I_a = I_b$

(d) $f_b = f_c$ and $I_b = I_c$

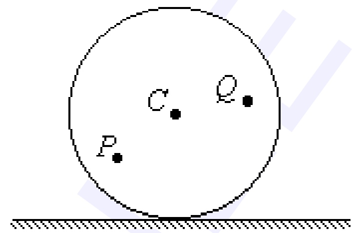
➢ The stopping potential for curves $a$ and $b$ is same.

$\therefore$ $f_a = f_b$

Also saturation current is proportional to intensity.

$\therefore \quad I_a < I_b$

**22.** A disc is rolling (without slipping) clockwise on a horizontal surface. $C$ is its center and $Q$ and $P$ are two points equidistant from $C$. Let $V_P$, $V_Q$ and $V_C$ be the magnitude of velocities of points $P$, $Q$ and $C$ respectively, then



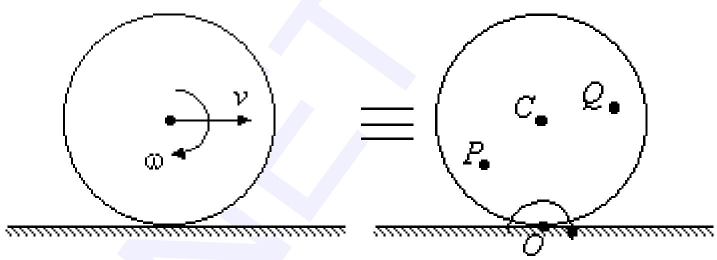

*(a) $V_Q > V_C > V_P$

(b) $V_Q < V_C < V_P$

(c) $V_Q = V_P,\ V_C = \frac{1}{2} V_P$

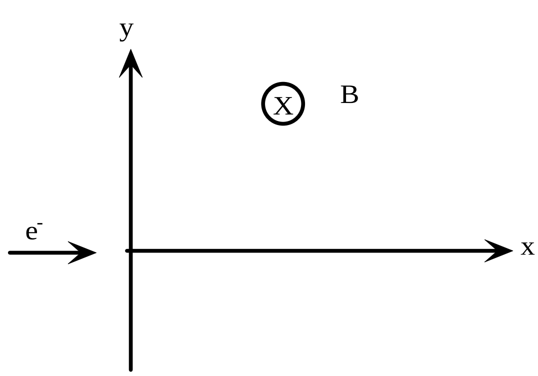
(d) $V_Q < V_C > V_P$

➢ For rolling without slipping




and distance $OP < OC < OQ$

**23.** An electron moving with a speed $u$ along the positive $x$-axis at $y = 0$ enters a region of uniform magnetic field $\vec{B} = -B_o\hat{k}$ which exists to the right of $y$-axis. The electron exits from the region after some time with the speed $v$ at ordinate $y$, then



(a) $v > u, y < 0$

(b) $v = u, y > 0$

(c) $v > u, y > 0$

*(d) $v = u, y < 0$

➢ The magnetic field will not do any work. Also, the force on the electron acts in such a way that it describes a circular path in clockwise direction.

**24.** A wire has a mass 0.3 ± 0.003 g, radius 0.5 ± 0.005 mm and length 6 ± 0.06 cm. The maximum percentage error in the measurement of its density is

(a) 1 (b) 2

(c) 3 *(d) 4

➢ Since $\rho = \frac{m}{\pi r^2 L}$

$$\left(\frac{\Delta\rho}{\rho}\right)\times 100 = \left(\frac{\Delta m}{m} + 2.\frac{\Delta r}{r} + \frac{\Delta L}{L}\right)\times 100 = 4$$

**25.** A small block slides without friction down an inclined plane starting from rest. Let $S_n$ be the distance travelled from time $t = n - 1$ to $t = n$. Then $\frac{S_n}{S_{n+1}}$ is

(a) $\frac{2n-1}{2n}$ (b) $\frac{2n+1}{2n-1}$

*(c) $\frac{2n-1}{2n+1}$ (d) $\frac{2n}{2n+1}$

➢ $S_t = u + \frac{1}{2}a(2t-1)$

**26.** A child is standing with folded hands at the center of a platform rotating about its central axis. The kinetic energy of the system is *K*. The child now stretches his arms so that the moment of inertia of the system doubles. The kinetic energy of the system now is

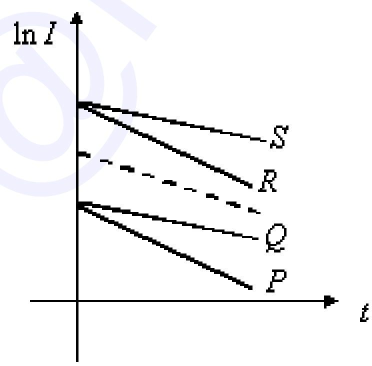
(a) $2K$ *(b) $\frac{K}{2}$

(c) $\frac{K}{4}$ (d) $4K$

➢ From COAM, the angular velocity of the system will become half.

As rotational kinetic energy = $\frac{1}{2}I\omega^2$, the kinetic energy will become half.

**27.** In an *RC* circuit while charging, the graph of $\ln I$ versus time is as shown by the dotted line in the adjoining diagram where *I* is the current. When the value of the resistance is doubled, which of the solid curves best represents the variation of ln *I* versus time?




(a) *P* *(b) *Q*

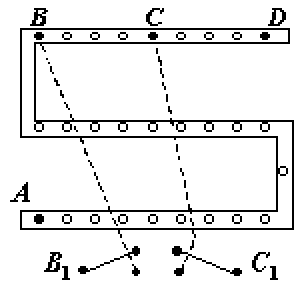
(c) *R* (d) *S*

➢ $I = \frac{E}{R}e^{-t/RC}$ for a charging RC circuit.

$$\therefore \quad \ln I = -\frac{t}{RC} + \ln\frac{E}{R}$$

When *R* is doubled, slope of the curve increases. Also, at *t* = 0 the current will be less for a increased value of resistance.

**28.** For the post office box arrangement to determine the value of unknown resistance, the unknown resistance should be connected between




(a) $B$ and $C$ (b) $C$ and $D$

*(c) $A$ and $D$ (d) $B_1$ and $C_1$

➢ $BC$, $CD$ and $BA$ are the known resistances. The unknown resistance is to be connected between $A$ and $D$.

## CHEMISTRY

**Break-up of questions:**

| Physical | Inorganic | Organic |
|---|---|---|
| 13 (47%) | 6 (21%) | 9 (32%) |

**Remarks: Physical + Organic constitute around 80% as expected.**

**1.** The pair of the compounds in which both the metals are in the highest possible oxidation state is

(a) $[Fe(CN)_6]^{3-}$, $[Co(CN)_6]^{3-}$ *(b) $CrO_2Cl_2$, $MnO_4^-$

(c) $TiO_3$, $MnO_2$ (d) $[Co(CN)_6]^{3-}$,$MnO_3$

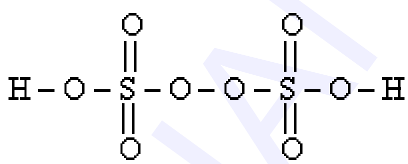
➢ $\overset{+6}{Cr}O_2Cl_2$ , $\overset{+7}{Mn}O_4^-$

**2.** The acid having O – O bond is

(a) $H_2S_2O_3$ (b) $H_2S_2O_6$

*(c) $H_2S_2O_8$ (d) $H_2S_4O_6$

➢



(Marshall's Acid)

**Remarks: This question was given in classroom test series for screening.**

**3.** $(NH_4)_2Cr_2O_7$ on heating liberates a gas. The same gas will be obtained by

* (a) heating $NH_4NO_2$ (b) heating $NH_4NO_3$

(c) treating $H_2O_2$ with $NaNO_2$ (d) treating $Mg_3N_2$ with $H_2O$

➢ $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + 4H_2O$

$NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$

**4.** Pb and Sn are extracted from their chief ore by

(a) Carbon reduction and self reduction *(b) Self reduction and carbon reduction

(c) Electrolysis and self reduction (d) Self reduction and electrolysis

➢ $PbS + 2PbO \longrightarrow 3Pb + SO_2$

$SnO_2 + C \longrightarrow Sn + CO_2$

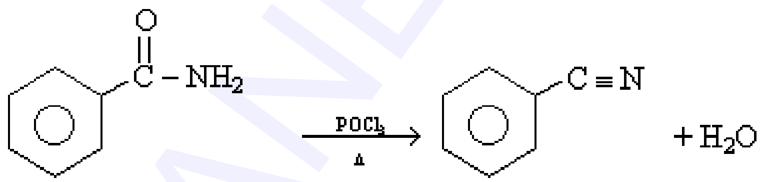
(Anthracite)

**5.** Benzamide on reaction with $POCl_3$ gives

(a) Aniline (b) Chlorobenzene

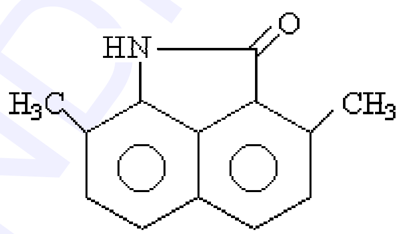
(c) Benzyl amine *(d) Benzonitrile

➢

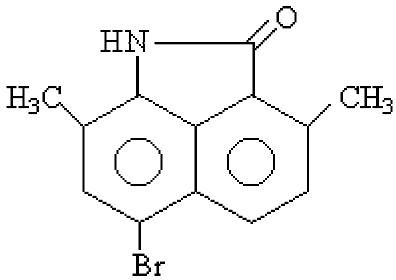


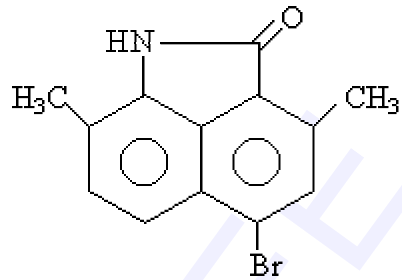

(dehydration)

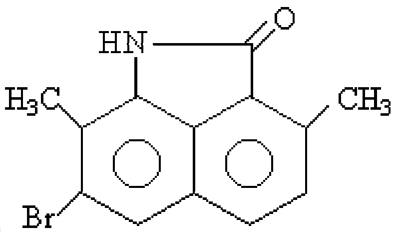
**6.** The major product obtained when $Br_2$/Fe is treated with

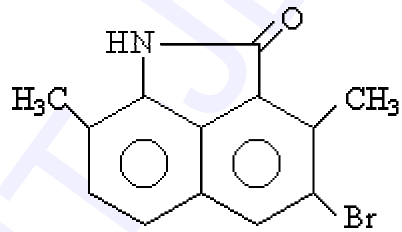


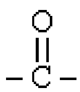

is

*(a) 

(b) 

(c) 

(d) 

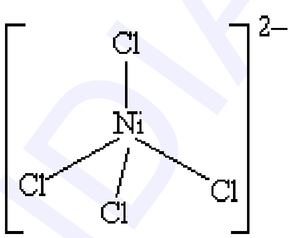
➢ The phenyl ring having H–N< group is activated while another one is deactivated due to , so electrophilic aromatic bromination will occur at para position with respect to H – N< group in activated ring.

**7.** The species having tetrahedral shape is

(a) $[PdCl_4]^{2-}$ (b) $[Ni(CN)_4]^{2-}$

(c) $[Pd(CN)_4]^{2-}$ *(d) $[NiCl_4]^{2-}$

➢



$sp^3$, Tetrahedral

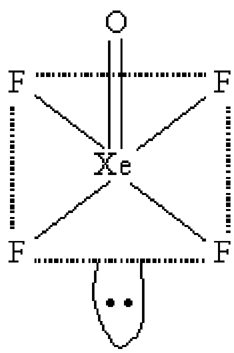
.

**8.** Total number of lone pair of electrons in $XeOF_4$ is

(a) 0 *(b) 1

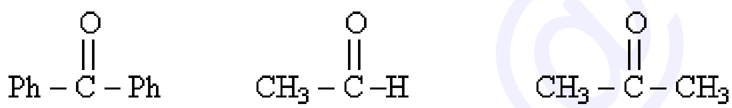
(c) 2 (d) 3

➢




Therefore, total number of lone pair of electron on central atom xenon = 1

**9.** The correct order of reactivity of PhMgBr with




(I) (II) (III)

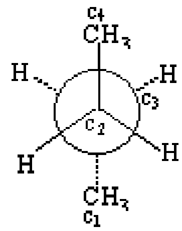
is

(a) (I) > (II) > (III) (b) (III) > (II) > (I)

*(c) (II) > (III) > (I) (d) (I) > (III) > (II)

➢ In nucleophilic addition reaction, the carbonyl compound will respond in preference, which is sterically more exposed and electronically have intact positive charge over carbonyl carbon. So reactivity order towards reaction with PhMgBr is (II) > (III) > (I).

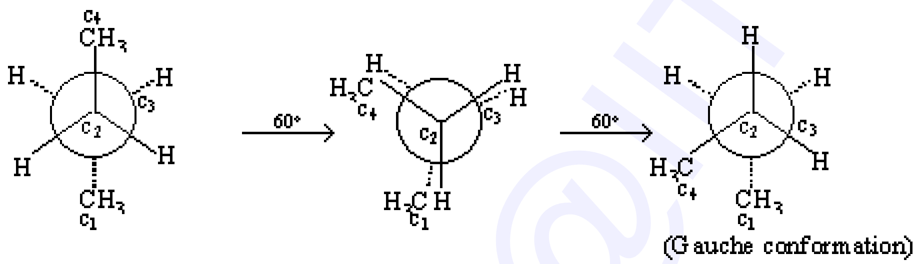
**10.** In the given conformation $C_2$ is rotated about $C_2 – C_3$ bond anticlockwise by an angle of 120° then the conformation obtained is




(a) Fully eclipsed conformation (b) Partially eclipsed conformation

*(c) Gauche conformation (d) Staggered conformation

➢




**11.** The spin magnetic moment of cobalt in the compound $Hg[Co(SCN)_4]$ is

(a) $\sqrt{3}$ (b) $\sqrt{8}$

*(c) $\sqrt{15}$ (d) $\sqrt{24}$

➢ In this complex $Co^{2+}$ ion have 3 unpaired electrons so spin only magnetic moment will be $\sqrt{3(3+2)}$ i.e. $\sqrt{15}$ BM.

**12.** A sodium salt on treatment with $MgCl_2$ gives white precipitate only on heating. The anion of the sodium salt is

*(a) $HCO_3^-$ (b) $CO_3^{2-}$

(c) $NO_3^-$ (d) $SO_4^{2-}$

➢ $MgCl_2 + 2NaHCO_3 \longrightarrow Mg(HCO_3)_2(aq) + 2NaCl$

$Mg(HCO_3)_2(aq) \xrightarrow{\Delta} MgCO_3\downarrow + H_2O + CO_2$

(white ppt.)

**13.** The product of oxidation of $I^-$ with $MnO_4^-$ in alkaline medium is

*(a) $IO_3^-$ (b) $I_2$

(c) $IO^-$ (d) $IO_4^-$

➤ $6MnO_4^- + I^- + 6OH^- \longrightarrow 6MnO_4^{2-} + IO_3^- + 3H_2O$

**14.** The emf of the cell

$Zn \mid Zn^{2+}\,(0.01\ M) \parallel Fe^{2+}\,(0.001\ M) \mid Fe$

at 298 K is 0.2905 then the value of equilibrium constant for the cell reaction is

(a) $e^{\frac{0.32}{0.0295}}$ * (b) $10^{\frac{0.32}{0.0295}}$

(c) $10^{\frac{0.26}{0.0295}}$ (d) $10^{\frac{0.32}{0.0591}}$

➤ Cell reaction: $Zn + Fe^{2+} \longrightarrow Zn^{2+} + Fe$

$$E = E° - \frac{0.0591}{2}\log\frac{10^{-2}}{10^{-3}}$$

$$E° = 0.2905 + \frac{0.0591}{2} = 0.32$$

$$0.32 = \frac{0.0591}{2}\log K_{eq}$$

$$K_{eq} = 10^{\frac{0.32}{0.0295}}$$

**15.** The pair of compounds in which both the compounds give positive test with Tollen's reagent is

(a) Glucose and Sucrose (b) Fructose and Sucrose

(c) Acetophenone and Hexanal *(d) Glucose and Fructose

➢ Tollen's reagent oxidizes the compound having aldehyde group like glucose and also oxidizes α–hydroxyketones having $-COCH_2OH$ group as in fructose.

**16.** According to molecular orbital theory which of the following statement about the magnetic character and bond order is correct regarding $O_2^+$.

(a) Paramagnetic and Bond order < $O_2$ *(b) Paramagnetic and Bond order > $O_2$

(c) Diamagnetic and Bond order < $O_2$ (d) Diamagnetic and Bond order > $O_2$

➢ $O_2$: $\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_x^2, \begin{cases}\pi 2p_y^2, \\ \pi 2p_z^2,\end{cases} \begin{cases}\pi^* 2p_y^1 \\ \pi^* 2p_z^1\end{cases}$; Bond order = $\frac{10-6}{2} = 2$

(two unpaired electrons in antibonding molecular orbital)

$O_2^+$: $\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_x^2, \begin{cases}\pi 2p_y^2, \\ \pi 2p_z^2,\end{cases} \begin{cases}\pi^* 2p_y^1 \\ \pi^* 2p_z^0\end{cases}$; Bond order = $\frac{10-5}{2} = 2.5$

(one unpaired electron in antibonding molecular orbital)

**17.** A weak acid HX has the dissociation constant $1 \times 10^{-5}$ M. It forms a salt NaX on reaction with alkali. The degree of hydrolysis of 0.1 M solution of NaX is

(a) 0.0001% *(b) 0.01%

(c) 0.1% (d) 0.15%

➢ $X^- + H_2O \rightleftharpoons HX + OH^-$

$K_h = \frac{10^{-14}}{10^{-5}}$

$$h = \sqrt{\frac{10^{-14}}{10^{-5} \times 0.1}} = 10^{-4}$$

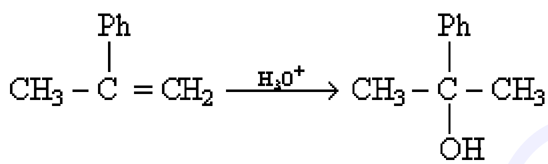
Percentage hydrolysis = 0.01%

**18.** The product of acid catalyzed hydration of 2–phenyl propene is

(a) 3–phenyl–2–propanol (b) 1–phenyl–2–propanol

*(c) 2–phenyl–2–propanol (d) 2–phenyl–1–propanol

➢




**19.** A 0.004 M solution of $Na_2SO_4$ is isotonic with a 0.010 M solution of glucose at same temperature. The apparent degree of dissociation of $Na_2SO_4$ is

(a) 25% (b) 50%

*(c) 75% (d) 85%

➢ $Na_2SO_4 \rightleftharpoons 2Na^+ + SO_4^{2-}$

$(0.004 - x)$ $2x$ $x$

Since both the solutions are isotonic

$0.004 + 2x = 0.01$

$\therefore$ $x = 3 \times 10^{-3}$

$\therefore$ Percentage dissociation = $\frac{3 \times 10^{-3}}{0.004} \times 100 = 75$

**20.** Adsorption of gases on solid surface is generally exothermic because

(a) Enthalpy is positive *(b) Entropy decreases

(c) Entropy increases (d) Free energy increases

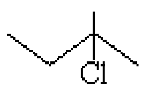
➢ When a gas is adsorbed on the surface the freedom of movement of its molecules becomes restricted. This decreases the randomness and hence the entropy of the gas decreases i.e., $\Delta S$ becomes negative.

**21.** On monochlorination of 2–methyl butane, the total number of chiral compounds is

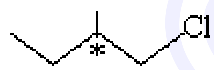
(a) 2 *(b) 4

(c) 6 (d) 8

➢ The possible monochlorinated products of 2–methyl butane are

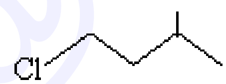



There is no chiral carbon

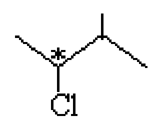



(+) and (–) forms

(2 compounds)




There is no chiral carbon

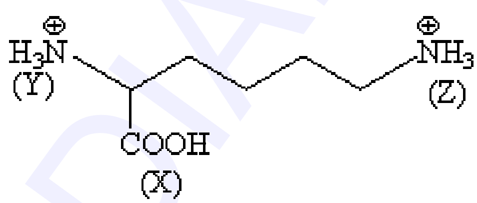



(+) and (–) forms

(2 compounds)

Therefore, a total of four chiral compounds are obtained.

**22.** In the compound given below




the correct order of the acidity of the positions (X), (Y) and (Z) is

(a) (Z) > (X) > (Y) *(b) (X) > (Y) > (Z)

(c) (X) > (Z) > (Y) (d) (Y) > (X) > (Z)

➢ Position (X) is obviously most acidic. Position (Y) is comparatively more acidic than that of (Z) due to the presence of electron withdrawing –COOH group in close proximity.

**23.** The root mean square velocity of one mole of a monoatomic gas having molar mass M is $U_{rms}$. The relation between the average kinetic energy (E) of the gas and $U_{rms}$ is

(a) $U_{rms} = \sqrt{\frac{3E}{2M}}$  (b) $U_{rms} = \sqrt{\frac{2E}{3M}}$

*(c) $U_{rms} = \sqrt{\frac{2E}{M}}$  (d) $U_{rms} = \sqrt{\frac{E}{3M}}$

➢ The root mean square velocity of one mole of a monoatomic gas is

$$U_{rms} = \sqrt{\frac{3RT}{M}}$$

and average kinetic energy (E) is $\frac{3}{2}RT$. The relation between (E) and $U_{rms}$ is

$$\sqrt{\frac{2E}{M}} = \sqrt{\frac{3RT}{M}}$$

$$U_{rms} = \sqrt{\frac{2E}{M}}$$

**24.** Two mole of an ideal gas is expanded isothermally and reversibly from 1 litre to 10 litre at 300 K. The enthalpy change (in kJ) for the process is

(a) 11.4 kJ  (b) –11.4 kJ

*(c) 0 kJ  (d) 4.8 kJ

➢ $\Delta H = nC_p\Delta T$

The process is isothermal therefore $\Delta T = 0$

$\therefore \quad \Delta H = 0$

**25.** The radius of which of the following orbit is same as that of the first Bohr's orbit of hydrogen atom?

(a) $He^{+}$ (n = 2)  (b) $Li^{2+}$ (n = 2)

(c) $Li^{2+}$ (n = 3)  *(d) $Be^{3+}$ (n = 2)

➢ $$r_H = 0.529\frac{n^2}{Z}\text{Å}$$

For hydrogen: n = 1 and Z = 1, Therefore $r_H = 0.529\text{Å}$

For $Be^{3+}$: Z = 4 and n = 2. Therefore, $r_{Be^{3+}} = \dfrac{0.529 \times 2^2}{4} = 0.529\text{Å}$

**26.** Which of the following used for the conversion of 2–hexyne into trans–2–hexane?

(a) $H_2/Pd/BaSO_4$  (b) $H_2$, $PtO_2$

(c) $NaBH_4$  *(d) $Li–NH_3/C_2H_5OH$`

➢ The partial reduction of alkynes by active metal in liquid ammonia takes place through trans vinylic anion, which ultimately produces trans alkene.

**27.** The reaction, **X** $\longrightarrow$ Product follows first order kinetics. In 40 minutes the concentration of **X** changes from 0.1 M to 0.025 M. Then the rate of reaction when concentration of **X** is 0.01 M

(a) $1.73 \times 10^{-4}$ M $min^{-1}$  (b) $3.47 \times 10^{-5}$ M $min^{-1}$

*(c) $3.47 \times 10^{-4}$ M $min^{-1}$  (d) $1.73 \times 10^{-5}$ M $min^{-1}$

➢ From data it is evident that

$$t_{1/2} = \frac{40}{2} = 20 \text{ min}$$

The rate of reaction when [X] is 0.01 M = k[X] = $\frac{0.693}{20} \times 0.01$ = $3.47 \times 10^{-4}$ M $min^{-1}$

**28.** The enthalpy of vapourization of a liquid is 30 kJ $mol^{-1}$ and entropy of vapourization is 75 J $mol^{-1}$ K. The boiling point of the liquid at 1 atm is

(a) 250 K *(b) 400 K

(c) 450 K (d) 600 K

➤ $dS = \frac{dQ_{rev.}}{T}$

$75 = \frac{30 \times 10^3}{T}$

T = 400 K

MATHEMATICS

**Break-up of questions:**

| Algebra | Trigonometry | Co-ordinate Geometry | Calculus | Vector/3D |
|---|---|---|---|---|
| 8 (28%) | 3 (11%) | 4 (14%) | 10 (36%) | 3 (11%) |

**Remarks: Calculus + Algebra constitute around 60% as expected.**

**1.** The angle between the tangents drawn from the point (1, 4) to the parabola $y^2 = 4x$ is

(a) $\pi/2$ *(b) $\pi/3$
(c) $\pi/4$ (d) $\pi/6$

➢ Any tangent to $y^2 = 4x$ is $y = mx + \frac{1}{m}$.

Since it passes through (1, 4), we have $4 = m + \frac{1}{m}$

$\Rightarrow \quad m^2 - 4m + 1 = 0$

$\Rightarrow \quad m_1 + m_2 = 4,\ m_1 . m_2 = 1$

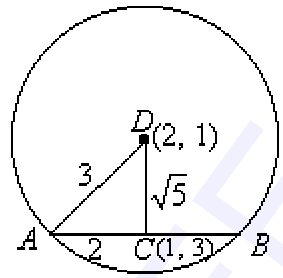
$\Rightarrow \quad |m_1 - m_2| = 2\sqrt{3}$

If θ is the required angle, then $\tan\theta = \frac{2\sqrt{3}}{1+1} = \sqrt{3} \Rightarrow \theta = \pi/3$

**2.** The radius of the circle, having centre at (2, 1), whose one of the chord is a diameter of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$

(a) 1 (b) 2
*(c) 3 (d) $\sqrt{3}$

➤ The centre $C$ of the given circle is (1, 3) and radius is 2. So $AB$, a diameter of the given circle has its mid point as (1, 3). The radius $DA$ of the required circle = 3.




**3.** If $\sin\left(\cot^{-1}(x+1)\right) = \cos\left(\tan^{-1} x\right)$, then $x =$

*(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$

(c) 0 (d) $\frac{9}{4}$

➤ $\sin\left(\cot^{-1}(x+1)\right) = \sin\left(\sin^{-1}\frac{1}{\sqrt{x^2+2x+2}}\right) = \frac{1}{\sqrt{x^2+2x+2}}$

$\cos\left(\tan^{-1} x\right) = \cos\left(\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right) = \frac{1}{\sqrt{1+x^2}}$

Thus $\frac{1}{\sqrt{x^2+2x+2}} = \frac{1}{\sqrt{1+x^2}} \Rightarrow x^2 + 2x + 2 = x^2 + 1 \Rightarrow x = -\frac{1}{2}$

**4.** $\int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx =$

(a) $\frac{\pi}{2}+1$ *(b) $\frac{\pi}{2}-1$

(c) $\pi$ (d) 1

➤ Let $x = \cos 2\theta$, $0 \le 2\theta \le \frac{\pi}{2}$.

Now $\sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}} \Rightarrow \int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx = 4\int_0^{\pi/4} \sin^2\theta\, d\theta = 2\int_0^{\pi/4} (1-\cos 2\theta)\,d\theta$

$= 2\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/4} = 2\left[\frac{\pi}{4} - \frac{1}{2}\right] = \frac{\pi}{2} - 1$

**5.** If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$, then α =

* (a) ± 3 (b) ± 2

(c) ± 5 (d) 0

➢ $125 = |A^3| = |A|^3 \Rightarrow |A| = 5 \Rightarrow \alpha^2 - 4 = 5$

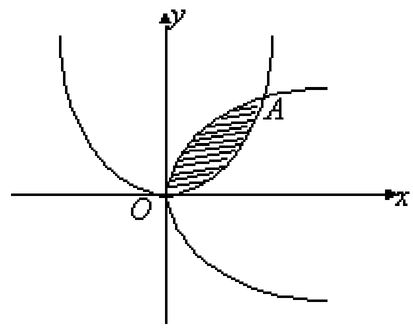
$\Rightarrow \quad \alpha = \pm 3$

**6.** If the area bounded by $y = ax^2$ and $x = ay^2$, $a > 0$, is 1, then $a$ =

(a) 1 *(b) $\frac{1}{\sqrt{3}}$

(c) $\frac{1}{3}$ (d) $-\frac{1}{\sqrt{3}}$

➢ The $x$ coordinate of $A$ is $\frac{1}{a}$.




According to the given condition

$1 = \int_0^{1/a} \left(\sqrt{\frac{x}{a}} - ax^2\right) dx$

$= \frac{1}{\sqrt{a}} \cdot \frac{2}{3}\left[x^{3/2}\right]_0^{1/a} - \frac{a}{3}\left[x^3\right]_0^{1/a}$

$\Rightarrow \quad a = \frac{1}{\sqrt{3}}$

**7.** $^{n-1}C_r = (K^2 - 3)\,.\,^{n}C_{r+1}$, if $K \in$

(a) $\left[-\sqrt{3}, \sqrt{3}\right]$ (b) $(-\infty, -2)$

(c) $(2, \infty)$ *(d) $(\sqrt{3}, 2]$

➤ We have $\dfrac{(n-1)!}{r!\,(n-r-1)!} = (K^2 - 3)\dfrac{n!}{(r+1)!\,(n-r-1)!}$, $0 \le r \le n-1$

$\Rightarrow$ $1 = (K^2 - 3)\dfrac{n}{r+1}$

$\Rightarrow$ $K^2 = \dfrac{r+1}{n} + 3,\ \dfrac{1}{n} \le \dfrac{r+1}{n} \le 1$

$\Rightarrow$ $K^2 \in \left[\dfrac{1}{n} + 3,\ 4\right],\ n \ge 2$

$\Rightarrow$ $K \in \left[-2, -\sqrt{\dfrac{1}{n} + 3}\right] \cup \left[\sqrt{\dfrac{1}{n} + 3},\ 2\right],\quad n \ge 2$

**8.** The first term of an infinite geometric progression is $x$ and its sum is 5. Then

(a) $0 \le x \le 10$ *(b) $0 < x < 10$

(c) $-10 < x < 0$ (d) $x > 10$

➤ According to the given conditions

$5 = \dfrac{x}{1-r}$, $r$ being the common ratio

$\Rightarrow$ $r = 1 - \dfrac{x}{5}$

As $|r| < 1$, $\left|1 - \dfrac{x}{5}\right| < 1$

$\Rightarrow \quad -1 < \frac{x}{5} - 1 < 1$

$\Rightarrow \quad 0 < x < 10$

**9**. If θ and φ are acute angles satisfying $\sin\theta = \frac{1}{2}$, $\cos\phi = \frac{1}{3}$, then θ + φ ∈

(a) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right]$ *(b) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

(c) $\left(\frac{2\pi}{3}, \frac{5\pi}{6}\right)$ (d) $\left(\frac{5\pi}{6}, \pi\right)$

➢ $\sin\theta = \frac{1}{2} \quad \Rightarrow \theta = \frac{\pi}{6}$

$\cos\phi = \frac{1}{3} \quad \Rightarrow \frac{\pi}{3} < \phi < \frac{\pi}{2}$

Thus $\frac{\pi}{2} < \theta + \phi < \frac{2\pi}{3}$

**10**. If $\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx} = -\cos x, y(0)$ = 1, then $y\left(\frac{\pi}{2}\right)$ =

(a) 1 (b) $\frac{1}{2}$

*(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

➢ The given differential equation is

$$\frac{\cos x}{2+\sin x}dx + \frac{dy}{y+1} = 0$$

$\Rightarrow \quad \ln(2+\sin x) + \ln(y+1) = \ln c$

$\Rightarrow \quad (y+1)(2+\sin x) = c$

⇒ $2 \times 2 = c \Rightarrow c = 4$.

Thus $y + 1 = \dfrac{4}{2 + \sin x}$

⇒ $y = \dfrac{2 - \sin x}{2 + \sin x}$

⇒ $y\left(\dfrac{\pi}{2}\right) = \dfrac{1}{3}$

**11.** If one root of the equation $x^2 + px + q = 0$ is the square of the other, then

(a) $p^3 + q^2 - q(3p + 1) = 0$
(b) $p^3 + q^2 + q(1 + 3p) = 0$
(c) $p^3 + q^2 + q(3p - 1) = 0$
*(d) $p^3 + q^2 + q(1 - 3p) = 0$

➢ Let $\alpha$ and $\alpha^2$ be the roots, then $\alpha + \alpha^2 = -p$, $\alpha^3 = q$.

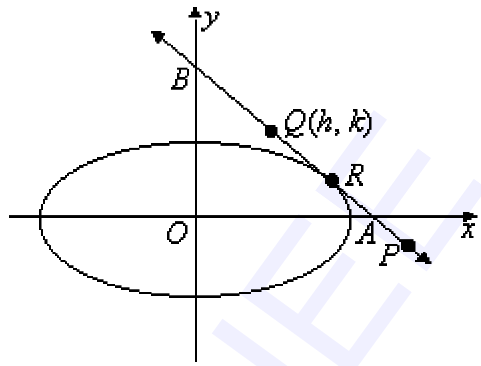
Now $(\alpha + \alpha^2)^3 = \alpha^3 + \alpha^6 + 3\alpha^3(\alpha + \alpha^2)$

⇒ $-p^3 = q + q^2 - 3pq$

⇒ $p^3 + q^2 + q(1 - 3p) = 0$

**12.** The locus of the middle point of the intercept of the tangents drawn from an external point to the ellipse $x^2 + 2y^2 = 2$, between the coordinates axes, is

(a) $\dfrac{1}{x^2} + \dfrac{1}{2y^2} = 1$
(b) $\dfrac{1}{4x^2} + \dfrac{1}{2y^2} = 1$
*(c) $\dfrac{1}{2x^2} + \dfrac{1}{4y^2} = 1$
(d) $\dfrac{1}{2x^2} + \dfrac{1}{y^2} = 1$




➢ Let the point of contact be $R \equiv (\sqrt{2}\cos\theta, \sin\theta)$.

Equation of tangent AB is $\frac{x}{\sqrt{2}}\cos\theta + y\sin\theta = 1$.

$\Rightarrow$ $A \equiv (\sqrt{2}\sec\theta, 0), B \equiv (0, \operatorname{cosec}\theta)$.

Let the middle point Q of AB be $(h, k)$.

$\Rightarrow$ $h = \frac{\sec\theta}{\sqrt{2}}, k = \frac{\operatorname{cosec}\theta}{2}$

$\Rightarrow$ $\cos\theta = \frac{1}{h\sqrt{2}}$, $\sin\theta = \frac{1}{2k}$

$\Rightarrow$ $\frac{1}{2h^2} + \frac{1}{4k^2} = 1$.

Thus required locus is $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$.

**13.** $\omega$ is an imaginary cube root of unity. If $(1 + \omega^2)^m = (1 + \omega^4)^m$, then least positive integral value of $m$ is

(a) 6 (b) 5

(c) 4 *(d) 3

➢ The given equation reduces to $(-\omega)^m = (-\omega^2)^m$

$\Rightarrow$ $\omega^m = 1 \Rightarrow m = 3$

**14.** If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{a}.\vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$, then $\vec{b} =$

* (a) $\hat{i}$ (b) $\hat{i} - \hat{j} + \hat{k}$

(c) $2\hat{j}-\hat{k}$ (d) $2\hat{i}$

➢ Let $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Now $\hat{j}-\hat{k} = \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ b_1 & b_2 & b_3 \end{vmatrix}$

$\Rightarrow$ $b_3 - b_2 = 0$, $b_1 - b_3 = 1$, $b_2 - b_1 = -1$

$\Rightarrow$ $b_3 = b_2$, $b_1 = b_2 + 1$

Now $\vec{a}.\vec{b} = 1$ $\Rightarrow b_1 + b_2 + b_3 = 1$ $\Rightarrow 3b_2 + 1 = 1 \Rightarrow b_2 = 0$

$\Rightarrow$ $b_1 = 1$, $b_3 = 0$

Thus $\vec{b} = \hat{i}$

**15.** The ratio of the sides of a triangle $ABC$ is $1 : \sqrt{3} : 2$. The ratio $A : B : C$ is

(a) 3 : 5 : 2 (b) $1 : \sqrt{3} : 2$

(c) 3 : 2 : 1 *(d) 1 : 2 : 3

➢ According to the given condition

$a = \lambda$, $b = \sqrt{3}\lambda$, $c = 2\lambda$, for some $\lambda \in R$.

Now $\cos A = \dfrac{3\lambda^2 + 4\lambda^2 - \lambda^2}{4\sqrt{3}\lambda^2} = \dfrac{6}{4\sqrt{3}} = \dfrac{\sqrt{3}}{2} \Rightarrow A = \dfrac{\pi}{6}$

$\cos B = \dfrac{\lambda^2 + 4\lambda^2 - 3\lambda^2}{4\lambda^2} = \dfrac{2\lambda^2}{4\lambda^2} = \dfrac{1}{2} \Rightarrow B = \dfrac{\pi}{3}$

Thus $C = \dfrac{\pi}{2}$

Therefore $A : B : C = 1 : 2 : 3$.

**16.** If $\int_0^{t^2} xf(x)dx = \frac{2t^5}{5}$, $t > 0$, then $f\left(\frac{4}{25}\right) =$

*(a) $\frac{2}{5}$ (b) $\frac{5}{2}$

(c) $-\frac{2}{5}$ (d) 1

➢ Given that $\int_0^{t^2} xf(x)dx = \frac{2t^5}{5}$

$\Rightarrow$ $2t \,.\, t^2 f(t^2) = 2t^4$

$\Rightarrow$ $f(t^2) = t$

$\Rightarrow$ $f\left(\frac{4}{25}\right) = \frac{2}{5}$, as $t > 0$

**17.** Let $f(x) = \sin x + \cos x$, $g(x) = x^2 - 1$. Thus $g(f(x))$ is invertible for $x \in$

(a) $\left[-\frac{\pi}{2}, 0\right]$ (b) $\left[-\frac{\pi}{2}, \pi\right]$

*(c) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ (d) $\left[0, \frac{\pi}{2}\right]$

➢ $g(f(x)) = g(\sin x + \cos x) = \sin 2x$, which is invertible for $x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

**18.** The area bounded by the angle bisectors of the lines $x^2 - y^2 + 2y = 1$ and the line $x + y = 3$, is

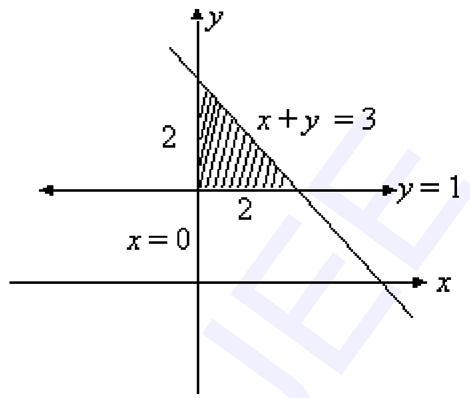
*(a) 2 sq. units (b) 3 sq. units

(c) 4 sq.units (d) 6 sq. units

➢ The angle bisectors of the lines given by

$x^2 - y^2 + 2y = 1$ are $x = 0$, $y = 1$.

The required area = $\frac{1}{2} \times 2 \times 2 = 2$




**19.** If $x^2 + 2ax + 10 - 3a > 0$ for all $x \in R$, then

*(a) $-5 < a < 2$ (b) $a < -5$
(c) $a > 5$ (d) $2 < a < 5$

➢ According to the given condition

$4a^2 - 4(10 - 3a) < 0$

$\Rightarrow \quad a^2 + 3a - 10 < 0$

$\Rightarrow \quad (a + 5)(a - 2) < 0$

$\Rightarrow \quad -5 < a < 2$

**20.** Let $f(x) = x^3 + bx^2 + cx + d$, $0 < b^2 < c$. Then $f$

(a) is bounded (b) has a local maxima
(c) has a local minima *(d) is strictly increasing

➢ $f'(x) = 3x^2 + 2bx + c$

Now its discriminant $= 4(b^2 - 3c)$

$= 4(b^2 - c) - 8c < 0$, as $b^2 < c$ and $c > 0$.

Therefore $f'(x) > 0$ for all $x \in R$.

Hence $f$ is strictly increasing.

**21.** The value of $\lambda$ for which the system of equations $2x - y - 2z = 2$, $x - 2y + z = -4$, $x + y + \lambda z = 4$ has no solution is

(a) 3 *(b) –3

(c) 2 (d) –2

➤ The coefficient determinant $= \begin{vmatrix} 2 & -1 & -2 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix}$

$= -3\lambda - 9$

For no solution the necessary condition is $-3\lambda - 9 = 0$

$\Rightarrow \quad \lambda = -3$

For $\lambda = -3$, there is no solution for the given system of equations.

**22.** Let $f(x) = \begin{cases} x^{\alpha} \ln x, & x > 0 \\ 0, & x = 0 \end{cases}$. Rolle's theorem is applicable to $f$ for $x \in [0, 1]$, if $\alpha =$

(a) –2 (b) –1

(c) 0 *(d) $\frac{1}{2}$

➤ For Rolle's theorem to be applicable to $f$, for $x \in [0, 1]$, we should have

(i) $f(1) = f(0)$,
(ii) $f$ is continuous for $x \in [0, 1]$ and $f$ is differentiable for $x \in (0, 1)$

From (i) $f(1) = 0$, which is true.

From (ii) $0 = f(0) = f(0+) = \lim_{x \to 0} x^{\alpha} . \ln x$, which is true only for positive values of $\alpha$. Thus (d) is correct.

**23.** The line $2x + \sqrt{6}\, y = 2$ is a tangent to the curve $x^2 - 2y^2 = 4$. The point of contact is

*(a) $(4, -\sqrt{6})$ (b) $(7, -2\sqrt{6})$

(c) (2, 3) (d) ( $\sqrt{6}$ , 1)

➢ Solving the line and the curve we get $x = 4$ and $y = -\sqrt{6}$ .

Thus point of contact is (4, $-\sqrt{6}$ ).

**24.** Three distinct numbers are selected from first 100 natural numbers. The probability that all the three numbers are divisible by 2 and 3 is

(a) $\frac{4}{25}$ (b) $\frac{4}{35}$

(c) $\frac{4}{55}$ *(d) $\frac{4}{1155}$

➢ The numbers should be divisible by 6. Thus the number of favourable ways is $^{16}C_3$ (as there are 16 numbers in first 100 natural numbers, divisible by 6).

Required probability is $\frac{^{16}C_3}{^{100}C_3} = \frac{16\times15\times14}{100\times99\times98} = \frac{4}{1155}$.

**25.** If $f$ is a strictly increasing function, then $\lim_{x\to0}\frac{f(x^2)-f(x)}{f(x)-f(0)} =$ is equal to

(a) 0 (b) 1

*(c) –1 (d) 2

➢ $\lim_{x\to0}\frac{f(x^2)-f(x)}{f(x)-f(0)} \quad \left(\frac{0}{0}\text{ form}\right)$

$= \lim_{x\to0}\frac{2xf'(x^2)-f'(x)}{f'(x)}$ (using L' Hospitals's rule)

$= -1+\lim_{x\to0}\frac{2xf'(x^2)}{f'(x)} = -1$, $f'(0) \neq 0$, as $f$ is strictly increasing.

**26.** If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then $k$ =

(a) $\frac{2}{9}$ *(b) $\frac{9}{2}$

(c) 0 (d) –1

➢ Any point on $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ is

$(2\lambda + 1, 3\lambda - 1, 4\lambda + 1), \lambda \in R.$

Any point on $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ is

$(\mu + 3, 2\mu + k, \mu), \mu \in R.$

The given lines intersect if and only if the system of equations (in $\lambda$ and $\mu$)

$2\lambda + 1 = \mu + 3$ (1)

$3\lambda - 1 = 2\mu + k$ (2)

$4\lambda + 1 = \mu$ (3)

has a unique solution.

Solving (1) and (3), we get $\lambda = -\frac{3}{2}$, $\mu = -5$.

From (2), we get $-\frac{9}{2} - 1 = -10 + k \Rightarrow k = \frac{9}{2}$.

**27.** If $\ln(x + y) = 2xy$, then $y'(0)$ =

*(a) 1 (b) –1

(c) 2 (d) 0

➢ $\ln(x + y) = 2xy$.

$\Rightarrow \quad \left(1+\frac{dy}{dx}\right)/(x+y) = 2\left(x.\frac{dy}{dx}+y\right)$

$\Rightarrow \quad \frac{dy}{dx} = \frac{1-2xy-2y^2}{2x^2+2xy-1}$

$\Rightarrow \quad y'(0) = \frac{1-2}{-1} = 1$, as at $x = 0$, $y = 1$.

**28.** A unit vector in the plane of the vectors $2\hat{i}+\hat{j}+\hat{k}$, $\hat{i}-\hat{j}+\hat{k}$ and orthogonal to $5\hat{i}+2\hat{j}+6\hat{k}$ is

(a) $\frac{6\hat{i}-5\hat{k}}{\sqrt{61}}$ *(b) $\frac{3\hat{j}-\hat{k}}{\sqrt{10}}$

(c) $\frac{2\hat{i}-5\hat{j}}{\sqrt{29}}$ (d) $\frac{2\hat{i}+\hat{j}-2\hat{k}}{3}$

➢ Let a unit vector in the plane of $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ be

$$\hat{a} = \alpha(2\hat{i}+\hat{j}+\hat{k})+\beta(\hat{i}-\hat{j}+\hat{k})$$

$$= (2\alpha+\beta)\hat{i}+(\alpha-\beta)\hat{j}+(\alpha+\beta)\hat{k}$$

As $\hat{a}$ is unit vector, we have

$$(2\alpha+\beta)^2+(\alpha-\beta)^2+(\alpha+\beta)^2 = 1$$

$\Rightarrow \quad 6\alpha^2+4\alpha\beta+3\beta^2 = 1 \qquad (1)$

As $\hat{a}$ is orthogonal to $5\hat{i}+2\hat{j}+6\hat{k}$, we get

$$5(2\alpha+\beta)+2(\alpha-\beta)+6(\alpha+\beta) = 0$$

$\Rightarrow \quad 18\alpha+9\beta = 0 \quad \Rightarrow \quad \beta = -2\alpha.$

From (1), we get $6\alpha^2-8\alpha^2+12\alpha^2 = 1$

$\Rightarrow \quad \alpha = \pm\frac{1}{\sqrt{10}} \qquad \Rightarrow \quad \beta = \mp\frac{2}{\sqrt{10}}$

Thus $\hat{a} = \pm\left(\frac{3}{\sqrt{10}}\hat{j} - \frac{1}{\sqrt{10}}\hat{k}\right)$

**Note: The questions are based on students' memory. INDIANET takes no responsibility in case of any discrepancy.**